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To construct the cubelet, we divide the above series into two sets of four numbers each, so that the sums of the two sets are equal, and the complementaries of one set are found in the other. This division is 0, 5, 6, 3 and 7, 2, 1, 4, which separates the complementaries and gives two sets, each amounting to 14. We can place one set in any desired order on one face, and it only remains to place the four complementaries in the opposite face, so that the four lines connecting complementary pairs are parallel.

These cubelets are arranged in the primary cube with the 0, 5, 6, 3 faces placed in the 1st, 3d, and 5th layers, and the 7, 2, 1, 4 faces placed in the 2d, 4th, and 6th layers, which arrangement satisfies the summations perpendicular to the layers.

It now remains to adjust the pairs in the cubelets to suit the summations in the layers and the four diagonals. We first arrange the pairs that will give the diagonal summations, and by doing so, we set the position of four numbers in each of the layers 3 and 4, and eight numbers in each of the layers 1, 2, 5 and 6. We then arrange the remaining numbers in the layers 1, 3 and 5 to suit the twelve summations of each layer, which consequently locates the numbers for layers 2, 4 and 6, since complementary pairs must lie perpendicularly to the cubes layers. This gives us a primary cube such as that shown in Fig. 4.

The numbers in each cell of Fig. 4 must then be multiplied by 27, and added to the respective cells in Fig. 3, which combination gives us the final cube shown in Fig. 1.

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MAGIC CUBE ON SIX.

FIRST OR TOP SQUARE.

106	8	7	212	209	109
199	116	113	16	12	195
196	114	115	11	15	200
21	203	202	103	100	22
17	205	208	99	104	18
112	5	6	210	211	107

SECOND SQUARE.

166	130	129	32	30	164
37	152	148	137	143	34
33	151	150	142	140	35
128	41	47	157	154	124
126	46	44	155	153	127
161	131	133	28	31	167

THIRD SQUARE.

163	135	136	25	27	165
36	145	149	144	138	39
40	146	147	139	141	38
121	48	42	156	159	125
123	43	45	158	160	122
168	134	132	29	26	162

FOURTH SQUARE.

55	192	191	83	81	49
93	60	57	176	174	91
89	62	63	172	175	90
182	74	77	70	65	183
180	75	73	68	71	184
52	188	190	82	85	54

FIFTH SQUARE.

50	185	186	86	88	56
92	61	64	169	171	94
96	59	58	173	170	95
179	79	76	67	72	178
181	78	80	69	66	177
53	189	187	87	84	51

SIXTH OR BOTTOM SQUARE.

111	1	2	213	216	108
194	117	120	9	13	198
197	119	118	14	10	193
20	206	207	98	101	19
24	204	201	102	97	23
105	4	3	215	214	110

In the cube, whose horizontal squares are here shown, the sum of each of the normal rows (those perpendicular to the faces of the cube) is 651, and the sum of each of the sixteen diagonals connecting the corners of the cube is the same.

These diagonals include the entire diagonals of the surfaces of the cube and the four diagonals of the solid running from corner to corner through the center of the cube.

DIAGONALS.

Top Square.	106	116	115	103	104	107
	109	12	11	202	205	112
Bottom Square.	111	117	118	98	97	110
	108	13	14	207	204	105

Front Square.	112	131	132	82	84	110
	107	31	29	190	189	105
Rear Square.	106	130	136	83	88	108
	109	30	25	191	185	111
Left Square.	106	37	40	182	181	105
	112	126	121	89	92	111
Right Square.	109	34	38	183	177	110
	107	127	125	90	94	108
Diagonals of the Solid.	106	152	147	70	66	110
	109	143	139	77	78	105
	107	153	156	63	61	111
	112	46	42	172	171	108

The foregoing cube was constructed in the following manner.

The foundation of this construction is the cube on 3 which is shown in the following squares.

FIRST OR TOP SQUARE

19	5	18
17	21	4
6	16	20

SECOND OR MIDDLE SQUARE.

15	25	2
1	14	27
26	3	13

THIRD OR BOTTOM SQUARE

8	12	22
24	7	11
10	23	9

The sum of each normal row in the above cube, whether running from left to right, from rear to front or from top to bottom, is 42; and the sum of each diagonal of which the central term 14 is a member, as 19 14 9, 5 14 23, 15 14 13, etc., is also 42.

Deduct 1 from each term of the above cube and multiply the remainder by 8. With each of these multiples construct a cubic group consisting of eight repetitions of the multiple. Substitute each of these groups for that term of the cube from which it was derived, and the result will be a cube with six terms in each row. The horizontal squares of this cube are shown in the following figures, the second square being the same as the first, the fourth as the third, and the sixth as the fifth.

BASIC CUBE.

FIRST, OR TOP, AND SECOND SQUARES.

144	144	32	32	136	136
144	144	32	32	136	136
128	128	160	160	24	24
128	128	160	160	24	24
40	40	120	120	152	152
40	40	120	120	152	152

THIRD AND FOURTH SQUARES.

112	112	192	192	8	8
112	112	192	192	8	8
0	0	104	104	208	208
0	0	104	104	208	208
200	200	16	16	96	96
200	200	16	16	96	96

FIFTH AND SIXTH SQUARES.

56	56	88	88	168	168
56	56	88	88	168	168
184	184	48	48	80	80
184	184	48	48	80	80
72	72	176	176	64	64
72	72	176	176	64	64

THE BASIC CUBE.

The sum of the terms in each normal row of the preceding cube is 624, and the sum of each diagonal which includes two terms from the central group of the cube is also 624. It follows that the middle two squares in each normal direction are magical and that each diagonal of the solid has the same sum as the normal rows. This cube is called the *basic* cube.

THE GROUP CUBE.

Another magic cube with six terms in each row was next constructed. This cube is called the *group* cube. Each position which in the basic cube is occupied by a cubic group of eight equal numbers is occupied in the group cube by a cubic group consisting of the numbers 1, 2, 3, 4, 5, 6, 7, 8. All of the rows and diagonals which have equal sums in the basic cube will have equal sums in the group cube.

THE GROUP CUBE.

FIRST OR TOP SQUARE.

3	2	8	6	5	3
5	1	4	7	2	8
8	7	3	5	3	1
4	6	8	2	2	5
5	3	3	2	8	6
2	8	1	5	7	4

SECOND SQUARE.

6	7	1	3	4	6
4	8	5	2	7	1
1	2	6	4	6	8
5	3	1	7	7	4
4	6	6	7	1	3
7	1	8	4	2	5

THIRD SQUARE.

3	2	4	8	7	3
1	4	7	3	4	8
7	8	2	5	1	4
6	5	8	3	3	2
8	5	1	2	8	3
2	3	5	6	4	7

FOURTH SQUARE.

6	7	5	1	2	6
8	5	2	6	5	1
2	1	7	4	8	5
3	4	1	6	6	7
1	4	8	7	1	6
7	6	4	3	5	2

FIFTH SQUARE.

2	3	8	7	2	5
8	5	4	6	3	1
2	1	2	8	8	6
3	5	5	3	4	7
8	6	5	1	2	5
4	7	3	2	8	3

SIXTH OR BOTTOM SQUARE.

7	6	1	2	7	4
1	4	5	3	6	8
7	8	7	1	1	3
6	4	4	6	5	2
1	3	4	8	7	4
5	2	6	7	1	6

THE COMPLETE CUBE.

Adding together the terms which occupy corresponding positions in the basic cube and the group cube the result is the complete cube shown below which contains the numbers from 1 to $6^3 = 216$.

THE COMPLETE CUBE.

FIRST OR TOP SQUARE.

147	146	40	38	141	139
149	145	36	39	138	144
136	135	163	165	27	25
132	134	168	162	26	29
45	43	123	122	160	158
42	48	121	125	159	156

SECOND SQUARE.

150	151	33	35	140	142
148	152	37	34	143	137
129	130	166	164	30	32
133	131	161	167	31	28
44	46	126	127	153	155
47	41	128	124	154	157

THIRD SQUARE.

115	114	196	200	15	11
113	116	199	195	12	16
7	8	106	109	209	212
6	5	112	107	211	210
208	205	17	18	104	99
202	203	21	22	100	103

FOURTH SQUARE.

118	119	197	193	10	14
120	117	194	198	13	9
2	1	111	108	216	213
3	4	105	110	214	215
201	204	24	23	97	102
207	206	20	19	101	98

FIFTH SQUARE.

58	59	96	95	170	173
64	61	92	94	171	169
186	185	50	56	88	86
187	189	53	51	84	87
80	78	181	177	66	69
76	79	179	178	72	67

SIXTH OR BOTTOM SQUARE.

63	62	89	90	175	172
57	60	93	91	174	176
191	192	55	49	81	83
190	188	52	54	85	82
73	75	180	184	71	68
77	74	182	183	65	70

THE FINAL CUBE.

In the complete cube, just given, the middle two squares in each direction are magical while the outer squares are not.

To bring these magical squares to the surface the squares of each set of parallel squares may be permuted as follows:

Original order 1, 2, 3, 4, 5, 6,
Permuted order ....3, 2, 1, 6, 5, 4.

The result is the final cube shown in the beginning of this article.

The above permutation is subject to two conditions. The several sets of parallel squares must all be permuted in the same manner. Any two parallel squares which in the original cube are located on opposite sides of the middle plane of the cube and at an equal distance from it, in the permuted cube must be located on opposite sides of the middle plane of the cube and at an equal distance from it. These conditions are for the protection of the diagonals.

John Worthington.

MAGIC IN THE FOURTH DIMENSION.

Definition of terms: *Row* is a general term; *rank* denotes a horizontal right-to-left row; *file* a row from front to back; and *column* a vertical row in a cube—not used of any horizontal dimension.

If n^2 numbers of a given series can be grouped so as to form a magic square and n such squares be so placed as to constitute a magic cube, why may we not go a step further and group n cubes in relations of the fourth dimension? In a magic square containing the natural series $1 \ldots n^2$ the summation is $\frac{n(n^2+1)}{2}$; in a magic cube with the series $1 \ldots n^3$ it is $\frac{n(n^3+1)}{2}$; and in an analogous fourth-dimension construction it naturally will be $\frac{n(n^4+1)}{2}$.

With this idea in mind I have made some experiments, and the results are interesting. The analogy with squares and cubes is not perfect, for rows of numbers can be arranged side by side to represent a visible square, squares can be piled one upon another to make a visible cube, but cubes cannot be so combined in drawing as to picture to the eye their higher relations. My expectation *a priori* was that some connection or relation, probably through some form of diagonal-of-diagonal, would be found to exist between the cubes containing the n^4 terms of a series. This particular feature did appear in the cases where n was odd. Here is how it worked out:

I. When n is odd.

1. Let $n=3$, then $S=123$.—The natural series $1 \ldots 81$ was divided into three sub-series such that the sum of each would be